D9/1 Consider elastic data traffic on a link between two routers at flow level. The traffic consists of TCP flows sharing the link, and arriving with intensity λ. The link capacity is denoted by C and the random flow size by L. In addition to the shared link, the rate of TCP flows is limited by access links. Let r denote the capacity of each access link.

(a) Consider this as an M/M/n-PS model. Assume that $\lambda = 80$ flows/s, $E[L] = 0.125 \cdot 10^6$ bytes, $C = 100$ Mbps, and $r = 10$ Mbps. Determine the throughput θ.

(b) What happens with the throughput if $C = 10$ Gbps?

D9/2 As in the previous problem, consider elastic data traffic on a link. Assume now that $\lambda = 80$ flows/s, $E[L] = 0.125 \cdot 10^6$ bytes, and $C = r = 100$ Mbps. In addition, there is an admission control scheme (to avoid overload situations) that allows at most 10 concurrent flows. Let $X(t)$ denote the number of flows sharing the link at time t, which is a Markov process.

(a) What is the traffic model in question (with Kendall's notation)?

(b) Draw the state transition diagram of $X(t)$.

(c) Derive the equilibrium distribution of $X(t)$.

(d) What is the mean number of flows sharing the link?

D9/3 Consider the M/M/1/3/3-PS model with $1/\nu = 1/\mu = 1$ (time unit). Determine the mean delay and the relative throughput of a customer.

D9/1 (a) This is a M/M/n-PS model with

$$n = \frac{C}{r} = \frac{100}{10} = 10$$

The throughput of a flow is thus (L9/25)

$$\theta = \frac{E[L]}{E[D]} = r \cdot \frac{n(1-\rho)}{p_W + n(1-\rho)},$$

where ρ refers to the load of the shared link (L9/15),

$$\rho = \frac{\lambda}{n\mu} = \frac{\lambda}{\frac{C}{E[L]}} = \frac{80}{\frac{100 \cdot 10^6}{8 \cdot 0.125 \cdot 10^6}} = \frac{4}{5} = 0.8,$$

and p_W to the following probability (L9/20, L8/31):

$$p_W = \frac{\beta}{\alpha+\beta} = \frac{\frac{(n\rho)^n}{n!(1-\rho)}}{\sum_0^{n-1} \frac{(n\rho)^i}{i!} + \frac{(n\rho)^n}{n!(1-\rho)}} = \cdots \overset{\text{num.}}{=} 0.41$$

Thus, the throughput is

$$\theta = r \cdot \frac{n(1-\rho)}{p_W + n(1-\rho)} = 10 \cdot \frac{2}{0.41+2} = 8.3\,\text{Mbps}$$

(b) As C becomes sufficiently large (while the offered traffic remains the same), the shared link hardly ever acts as a bottleneck so that the throughputs approach the access rate $r = 10$ Mbps. Thus, we deduce that, for $C = 10$ Gbps,

$$\theta \approx r = 10\,\text{Mbps}$$

D9/2 (a) Now

$$n = \frac{C}{r} = \frac{100}{100} = 1.$$

In addition, the admission control limits the number of concurrent flows up to 10. So this is a M/M/1/10-PS model with parameters $\lambda = 80$ flows/s and $\mu = r/E[L] = 100$ flows/s.

(b) Figure 1.

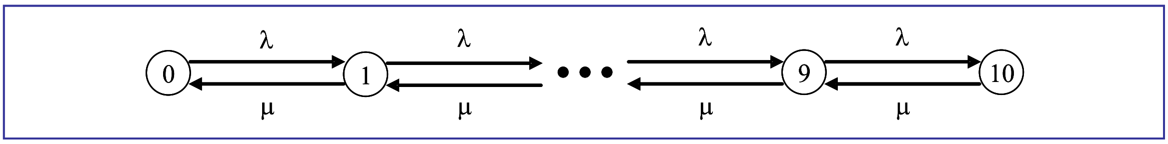

Figure 1: [D9/2] State transition diagram.

(c) We see from Figure 1 that $X(t)$ is an irreducible birth-death-process (L6/16). Since the state space is finite, the equilibrium distribution π exists, and it can be derived based on the local balance equations (LBE) and the normalization condition (N), cf. L6/17.

Let us start with the LBE's for states $i-1$ and i, where $i = 1, \ldots, N$:

$$\pi_{i-1}\lambda = \pi_i \mu$$

This results in the following recursion:

$$\pi_i = \pi_{i-1}(\frac{\lambda}{\mu}) = \pi_{i-2}(\frac{\lambda}{\mu})^2 = \ldots = \pi_0(\frac{\lambda}{\mu})^i = \pi_0\, \rho^i,$$

where $\rho = \lambda/\mu = 80/100 = 0.8$. The remaining probability π_0 is determined by (N):

$$\pi_0 + \pi_1 + \ldots + \pi_{10} = \pi_0 \sum_{i=0}^{10} \rho^i = \pi_0 \frac{1-\rho^{11}}{1-\rho} = 1$$

Thus,

$$\pi_0 = \frac{1-\rho}{1-\rho^{11}} = \frac{0.2}{1-(0.8)^{11}} \overset{\text{num.}}{=} 0.219$$

Thus, the equilibrium distribution is the truncated geometric distribution:

$$\begin{aligned} \pi &= (\pi_i \mid i = 0, 1, \ldots, 10) \\ &= (0.219, 0.175, 0.140, 0.112, 0.090, 0.072, 0.057, 0.046, 0.0370.029, 0.023) \end{aligned}$$

(d) The mean number of flows sharing the link is

$$E[X] = \sum_{i=0}^{10} i \cdot \pi_i = \ldots \overset{\text{num.}}{=} 2.97$$

D9/3 By L9/31, the equilibrium distribution of the M/M/1/k/k model reads as follows:

$$\pi_i = \frac{\frac{(\frac{\nu}{\mu})^i}{(k-i)!}}{\sum_{j=0}^{k} \frac{(\frac{\nu}{\mu})^j}{(k-j)!}}.$$

For $k = 3$ and $\nu = \mu = 1$, we get

$$\begin{aligned} \pi_0 &= \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{2}+1+1} = \frac{1}{1+3+6+6} = \frac{1}{16} = 0.062 \\ \pi_1 &= \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{2}+1+1} = \frac{3}{1+3+6+6} = \frac{3}{16} = 0.187 \\ \pi_2 &= \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{2}+1+1} = \frac{6}{1+3+6+6} = \frac{6}{16} = 0.375 \\ \pi_3 &= \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{2}+1+1} = \frac{6}{1+3+6+6} = \frac{6}{16} = 0.375 \end{aligned}$$

Thus, the mean number of customers in the system is

$$E[X] = \sum_{i=1}^{3} i \cdot \pi_i = \frac{3}{16} + 2 \cdot \frac{6}{16} + 3 \cdot \frac{6}{16} = \frac{33}{16} = 2.062$$

To apply Little's formula, we also need to know the rate λ at which new customers enter the system. When in state $X = i$, which happens with probability π_i, new customers enter the system with rate $(3 - i)\nu$. Thus, for $\nu = 1$, we get

$$\lambda = \sum_{i=0}^{3} (3 - i)\nu \cdot \pi_i = 3 \cdot \frac{1}{16} + 2 \cdot \frac{3}{16} + \frac{6}{16} = \frac{15}{16} = 0.937$$

Now, by Little's formula (L1/31), the mean delay is

$$E[D] = \frac{E[X]}{\lambda} = \frac{11}{5} = 2.2$$

On the other hand, the mean service time is $E[S] = 1/\mu = 1$ so that the relative throughput (cf. L9/22) is

$$\frac{E[S]}{E[D]} = \frac{1}{\frac{11}{5}} = \frac{5}{11} = 0.454$$