TKK HELSINKI UNIVERSITY OF TECHNOLOGY
Department of Communications and Networking
S-38.1145 Introduction to Teletraffic Theory, Spring 2008

Demonstrations
Lecture 5
31.1.2008

D5/1 Buses are leaving from a bus stop regularly in every 15 minutes. Taxis pass by according to a Poisson process with rate once in 15 minutes. You arrive at the bus stop at a random time instant.

(a) What is your expected waiting time until the first bus arrives?
(b) What is your expected waiting time until the first taxi arrives?
(c) What is the probability that you have to wait longer than 10 minutes until the first bus or taxi arrives?

D5/2 Solve the previous problem by assuming now that you arrive at the bus stop just after a bus has left it.

D5/3 Connection requests arrive at a server according to a Poisson process with intensity λ. If the server is overloaded, its throughput collapses quickly. To prevent such incidents, the server implements a congestion control scheme based on gapping. In this scheme after every accepted request the server refuses to accept any new connections during the following time period of length T. Assume that requests arriving during the gap interval are just discarded and they are not renewed.

a) How many requests are accepted in a time unit on average?
b) Determine the rate of accepted requests in the extreme cases when T is very large or very small, respectively.

D5/1 (a) Since the interarrival times between the bus arrivals are constant (15 min), the waiting time T_{b} from a random time instant to the arrival of the next bus is uniformly distributed between 0 and 15 minutes. In other words, $T_{\mathrm{b}} \sim U(0, 15)$ with mean equal to $15/2 = 7.5$ min (L4/41). Thus, you have to wait for a bus on average 7.5 min.

(b) The interarrival times between the arriving taxis are exponentially distributed with mean 15 min. By the memoryless property (L4/43), the waiting time T_{t} from a random time instant to the arrival of the next taxi is also exponentially distributed with mean 15 min. Thus, you have to wait for a taxi on average 15 min.

(c) Let $T = \min\{T_{\mathrm{b}}, T_{\mathrm{t}}\}$ denote the waiting time until the next bus or taxi arrives. Since T_{b} and T_{t} are independent and

$$P\{T_{\mathrm{b}} > 10\} \overset{\text{L4/41}}{=} \frac{15-10}{15-0} = \frac{1}{3}, \quad P\{T_{\mathrm{t}} > 10\} \overset{\text{L4/42}}{=} e^{-\frac{10}{15}} = e^{-\frac{2}{3}},$$

the probability asked is

$$P\{T > 10\} \overset{\text{L4/15}}{=} P\{T_{\mathrm{b}} > 10\}P\{T_{\mathrm{t}} > 10\} = \frac{1}{3} \cdot e^{-\frac{2}{3}} \overset{\text{num.}}{=} 0.17$$

D5/2 (a) If you arrive just after the departure of the previous bus, your waiting time T_{b}° is exactly the same as the interarrival between two buses. Thus, you have to wait for the next bus (exactly) 15 min.

(b) But since the buses and the taxis arrive independently of each others, your waiting time until the arrival of the next taxi is still exponentially distributed with mean 15 min. Thus, you have to wait for the next taxi on average 15 min.

(c) Let $T = \min\{T_{\mathrm{b}}^{\circ}, T_{\mathrm{t}}\}$ denote the waiting time until the next bus or taxi arrives. Since T_{b}° and T_{t} are independent and $T_{\mathrm{b}}^{\circ} = 15 > 10$ with probability 1, we get

$$P\{T > 10\} \overset{\text{L4/15}}{=} P\{T_{\mathrm{b}}^{\circ} > 10\}P\{T_{\mathrm{t}} > 10\} = 1 \cdot e^{-\frac{2}{3}} \overset{\text{num.}}{=} 0.51$$

D5/3 (a) The interval between two accepted requests is $t + I$, where the remaining interarrival time (between *any* two requests) is due to the memoryless property (L4/43) exponentially distributed with mean $1/\lambda$. Thus, the average interval between two accepted requests is $t + 1/\lambda$ so that $1/(t + 1/\lambda)$ requests are accepted in a time unit on average.

(b) As t shrinks to zero, the acceptance rate approaches λ. But when t increases without limits, the acceptance rate is approximately $1/t$, which, for one, becomes negligibly small.