D1/1 Consider a pure loss system with 10 servers. The average service time is 3 min. It is also known that the average number of customers in the system is 7.0 and an arriving customer is lost with probability of 14%. Determine the total arrival rate of new customers (including both those who enter the system and those who are lost)?

D1/2 Consider a pure single server queueing system with average service rate of 2.5 customers/s. New customers arrive at rate 2.0 customers/s. The average total delay is 3.0 s (including both the waiting time and the service time).

(a) What is the average number of customers in the whole system?

(b) What is the average number of waiting customers?

(c) What is the average number of customers in service?

(d) What is the average number of departing customers during an interval of length 10 s?

D1/3 According to measurements, there are, on average, 1000 packets in a subnetwork. This subnetwork is connected with the rest of the trunk network via four nodes. The arrival rates of packets (coming from the rest) to these four nodes are $\lambda_1 = 200$, $\lambda_2 = 300$, $\lambda_3 = 400$, and $\lambda_4 = 500$ packets/s, respectively. Determine the average time that a packet spends in the subnetwork.

D1/1 The average number of customers in the system is $\bar{N} = 7.0$ and each customer's average delay is $\bar{T} = 3.0$ minutes. According to Little's formula (L1/31), new customers enter the system at rate

$$\lambda = \frac{\bar{N}}{\bar{T}} = \frac{7.0}{3.0} = 2.33 \text{ customers/min.}$$

Since the fraction of lost customers is $p_{\text{loss}} = 0.14$, the total arrival rate is

$$\lambda_{\text{tot}} = \frac{\lambda}{1 - p_{\text{loss}}} = \frac{7.0}{3.0 \cdot (1 - 0.14)} = 2.71 \text{ customers/min.}$$

D1/2 (a) Let us first apply Little's formula to the whole system. New customers arrival at rate $\lambda = 2.0$ customers/s and the average total delay is $\bar{T} = 3.0$ s. Thus, by Little's formula (L1/31), the average number of customers in the system is

$$\bar{N} = \lambda \bar{T} = 2.0 \cdot 3.0 = 6.0 \text{ customers.}$$

(b) Let us now apply Little's formula to the subsystem of waiting customers. The arrival rate into this subsystem is also $\lambda = 2.0$ customers/s. On the other hand, we know that the average service time is $\bar{T}_{\text{S}} = 1/2.5 = 0.4$ s. Thus, the average waiting time is the difference $\bar{T}_{\text{W}} = \bar{T} - \bar{T}_{\text{S}} = 3.0 - 0.4 = 2.6$ s. According to Little's formula (L1/31), the number of waiting customers is

$$\bar{N}_{\text{W}} = \lambda \bar{T}_{\text{W}} = 2.0 \cdot 2.6 = 5.2 \text{ customers.}$$

(c) Let us finally apply Little's formula to the subsystem of customers in service. Since no customers are lost, the arrival rate into this subsystem is also $\lambda =$ 2.0 customers/s. In addition, we know that the average delay in this subsystem is the same as the average service time, $\bar{T}_{\text{S}} = 1/2.5 = 0.4$ s. Thus, by Little's formula (L1/31), the average number of customers in service is

$$\bar{N}_{\text{S}} = \lambda \bar{T}_{\text{S}} = 2.0 \cdot 0.4 = 0.8 \text{ customers.}$$

Of course, this could have been concluded more directly from the reults of (a) and (b) as follows:

$$\bar{N}_{\text{S}} = \bar{N} - \bar{N}_{\text{W}} = 6.0 - 5.2 = 0.8 \text{ customers.}$$

(d) Since the arrival rate ($\lambda = 2.0$ customers/s) is smaller than the service rate ($\mu =$ 2.5 asiakasta/s), the system is stable. Thus, the departure rate is the same as the arrival rate $\lambda = 2.0$ customers/s. It follows that, during an interval of length $\Delta = 10$ s, the average number of departing customers is

$$\bar{N}_{\Delta} = \lambda \Delta = 2.0 \cdot 10 = 20.0 \text{ customers.}$$

D1/3 The total packet arrival rate into the subnetwork is

$$\lambda = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 200 + 300 + 400 + 500 = 1400 \text{ packets/s.}$$

In addition, we know that the average number of packets in the subnetwork is

$$\bar{N} = 1000 \text{ packets.}$$

Thus, by Little's formula (L1/31), the average time that a packet spends in the subnetwork is

$$\bar{T} = \frac{\bar{N}}{\lambda} = \frac{1000}{1400} = 0.71 \text{ s.}$$