Exercise 6
27.2.2007

Problems 2, 3, and 6 are homework exercises. Return your answers into the course box of the laboratory (G-wing, 2. floor) latest at 10.00 on Tuesday 27.2.

1. *Demo*
   Simulate, according to the discrete event simulation principles presented in the lectures, the evolution of the queue length process $Q(t)$ (those waiting to be serviced and the one being serviced) in an M/M/1-FIFO queue during the interval $[0, T]$ assuming that the system is empty in the beginning ($Q(0) = 0$). Let $\lambda = 1/2$, $\mu = 1$, and $T = 2000$. Make $n = 100$ independent simulation runs. Independent means that the seed value for the random number generation changes. In each simulation run, calculate the mean queue length X in the interval $[T_0, T]$, where $T_0 = 1000$, from the equation

   $$X = \frac{1}{T - T_0} \int_{T_0}^{T} Q(t)dt.$$

   By this way, you get n IID samples $X_1, X_2, \ldots, X_n$ of the mean queue length in this interval.

   (a) Calculate and plot the sample average $\bar{X}_m$, for $m = 10, 20, \ldots, n$,

   $$\bar{X}_m = \frac{1}{m} \sum_{i=1}^{m} X_i.$$

   (b) Calculate and plot the square root of the sample variance, S_m, for $m = 10, 20, \ldots, n$,

   $$S_m = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} (X_i - \bar{X}_m)^2}.$$

   (c) Calculate and plot the confidence interval for the sample average $\bar{X}_m$ at confidence level 95% for $m = 10, 20, \ldots, n$, assuming that the samples are IID and from a normal distribution, but with an unknown variance.

2. *Homework exercise*
   Generation of random numbers.

   (a) Generate 4 (pseudo) random numbers from U$(0, 1)$ distribution using the MCG algorithm presented in the lecture (Slide 11/25) with parameters $m = 2^{31} - 1$, $a = 16807$ ja $Z_0 = 920107$.

   (b) Utilizing the random numbers generated in part (a), generate 4 random numbers from each of the following distributions: U$(1, 2)$, Geom(0.5), and Exp(2). Use the methods described in the lectures.

3. *Homework exercise*
   Confidence intervals.

   (a) Assume that the simulation runs have yielded the following independent observations X_i for a performance parameter α: 2.47, 5.32, 3.63, 4.16, 2.40, 6.07. Calculate the 95% confidence interval for parameter α assuming that the variance is known ($D^2[X_i] = 2$).

(b) Assume that we have simulated an M/M/1 queueing system and we have measured the waiting time W_n of N consecutive customers, i.e., W_n denotes the waiting time of the nth customer. Based on the samples W_n, we can easily compute an estimate of the mean waiting time, $\bar{W} = \frac{1}{N}\sum_{n=1}^{N} W_n$, but can one use the samples W_n to estimate the confidence interval and if not then why not?

4. *Demo*
Consider the following network with 4 nodes and 10 links. The set of nodes is denoted by $\mathcal{N} = \{\mathrm{a, b, c, d}\}$, and the set of links by $\mathcal{J} = \{1, 2, \ldots, 10\}$. The properties of various links are given in the table below (j = link index, n_j = origin node, m_j = destination node, c_j = link capacity).

j	n_j	m_j	c_j
1	a	b	10
2	b	a	10
3	a	c	10
4	c	a	10
5	a	d	10
6	d	a	10
7	b	c	4
8	c	b	4
9	c	d	4
10	d	c	4

Draw the network topology. What is the number of OD pairs? What is the total number of paths? What is the total number of shortest paths assumed that the links have unit weights ($w_j = 1$ for all j)?

5. *Demo*
Consider again the network specified in the previous problem. The network is loaded by the traffic demands given by the traffic matrix

$$\mathbf{T} = \begin{pmatrix} 0 & 5 & 5 & 5 \\ 5 & 0 & 2 & 2 \\ 5 & 2 & 0 & 2 \\ 5 & 2 & 2 & 0 \end{pmatrix}.$$

(a) From this information, formulate a Load Balancing Problem given in the lecture (Slide 12/25), and give the optimal solution with confirming arguments.

(b) Determine the link loads resulting from this optimal routing scheme.

6. *Homework exercise*
Consider again the network and traffic specified in the previous problems. Assume now that the shortest path alogorithm with unit weights ($w_j = 1$ for all j) is applied (instead of optimal routing) together with the ECMP principle presented on Slide 12/17.

(a) Determine the link loads resulting from this shortest path routing routing scheme.

(b) Give a better routing scheme achieved by modifying the link weights.