HELSINKI UNIVERSITY OF TECHNOLOGY
Networking Laboratory
S-38.1145 Introduction to Teletraffic Theory, Spring 2007

Exercise 3
6.2.2007

Problems 2, 5 and 6 are homework exercises. Return your answers into the course box of the laboratory (G-wing, 2. floor) latest at 10.00 on Tuesday 6.2.

1. *Demo*
   Buses are leaving from a bus stop regularly in every 15 minutes. Taxis pass by according to a Poisson process with rate once in 15 minutes. You arrive at the bus stop at a random time instant.

   (a) What is your expected waiting time until the first bus arrives?

   (b) What is your expected waiting time until the first taxi arrives?

   (c) What is the probability that you have to wait longer than 10 minutes before the first taxi or bus passes by?

2. *Homework exercise*
   A link in a packet switched network carries on average 1 packet/ms. Assume that packets arrive according to a Poisson process. Each packet is a data packet with probability 0.9 and an acknowledgement (ACK) with probability 0.1 independently of each other. Consider a random time interval of length one millisecond (ms).

   a) What is the probability that there are exactly two data packets and no ACKs on the link during the interval?

   b) Assume now that two packets have been observed during the interval. What is the probability that they both are data packets?

3. *Demo*
   Connection requests arrive at a server according to a Poisson process with intensity λ. If the server is overloaded, its throughput collapses quickly. To prevent such incidents, the server implements a congestion control scheme based on gapping. In this scheme after every accepted request the server refuses to accept any new connections during the following time period of length T. Assume that requests arriving during the gap interval are just discarded and they are not renewed.

   a) How many requests are accepted in a time unit on average?

   b) Determine the rate of accepted requests in the extreme cases when T is very large or very small, respectively.

4. *Demo*
   The Ehrenfest model was used, at a time, to shed light on a paradox related to the second law of thermodynamics. The model can be described as follows. A closed system is constructed from K randomly moving gas molecules and two containers which are connected so that each of the molecules changes the container at rate λ, independently of the other molecules. Let $X(t)$ denote the number of molecules in one of the containers.

   a) Draw the state transition diagram of Markov process $X(t)$.

   b) Derive the equilibrium distribution of $X(t)$.

   c) Compare the conditional probabilities $P\{X(t) = K \mid X(0) = \frac{K}{2}\}$ and $P\{X(t) = \frac{K}{2} \mid X(0) = K\}$ (K assumed to be even), when t is large.

5. *Homework exercise*

A Markov process is defined in the state space $\{0, 1, 2, 3\}$ with the state transitions rates q_{ij} collected in the transition matrix $Q = (q_{ij} \mid i, j = 0, 1, 2, 3)$, where $q_{ii} = -q_i$ for all i, as follows:

$$Q = \begin{pmatrix} -4 & 1 & 0 & 3 \\ 3 & -4 & 1 & 0 \\ 0 & 3 & -4 & 1 \\ 1 & 0 & 3 & -4 \end{pmatrix}$$

(a) Draw the state transition diagram of Markov process $X(t)$.

(b) Derive the equilibrium distribution of $X(t)$.

6. *Homework exercise*

A Markov process is defined in the state space $\{0, 1, 2, 3\}$ with the state transitions rates q_{ij} collected in the transition matrix $Q = (q_{ij} \mid i, j = 0, 1, 2, 3)$, where $q_{ii} = -q_i$ for all i, as follows:

$$Q = \begin{pmatrix} -3 & 1 & 1 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

(a) Draw the state transition diagram of Markov process $X(t)$.

(b) Derive the equilibrium distribution of $X(t)$.