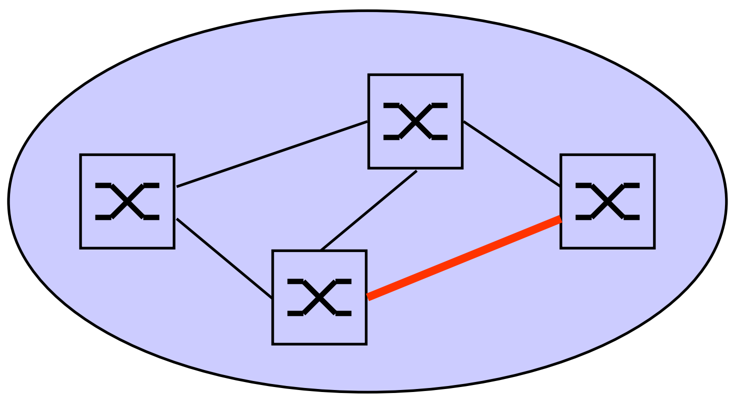
3. Examples

Contents

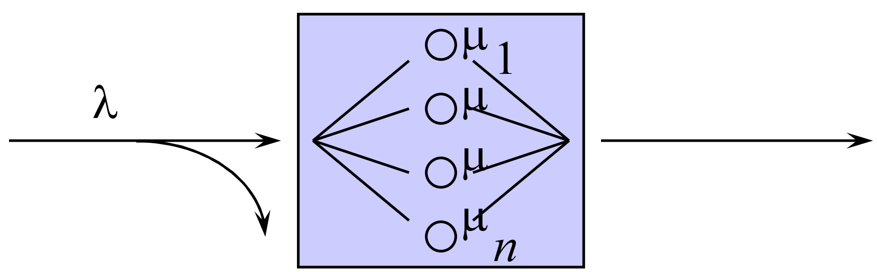
Classical model for telephone traffic (1)

- **Loss models** have traditionally been used to describe (circuit-switched) telephone networks
  - Pioneering work made by Danish mathematician A.K. Erlang (1878-1929)
- Consider a link between two telephone exchanges
  - traffic consists of the ongoing telephone calls on the link

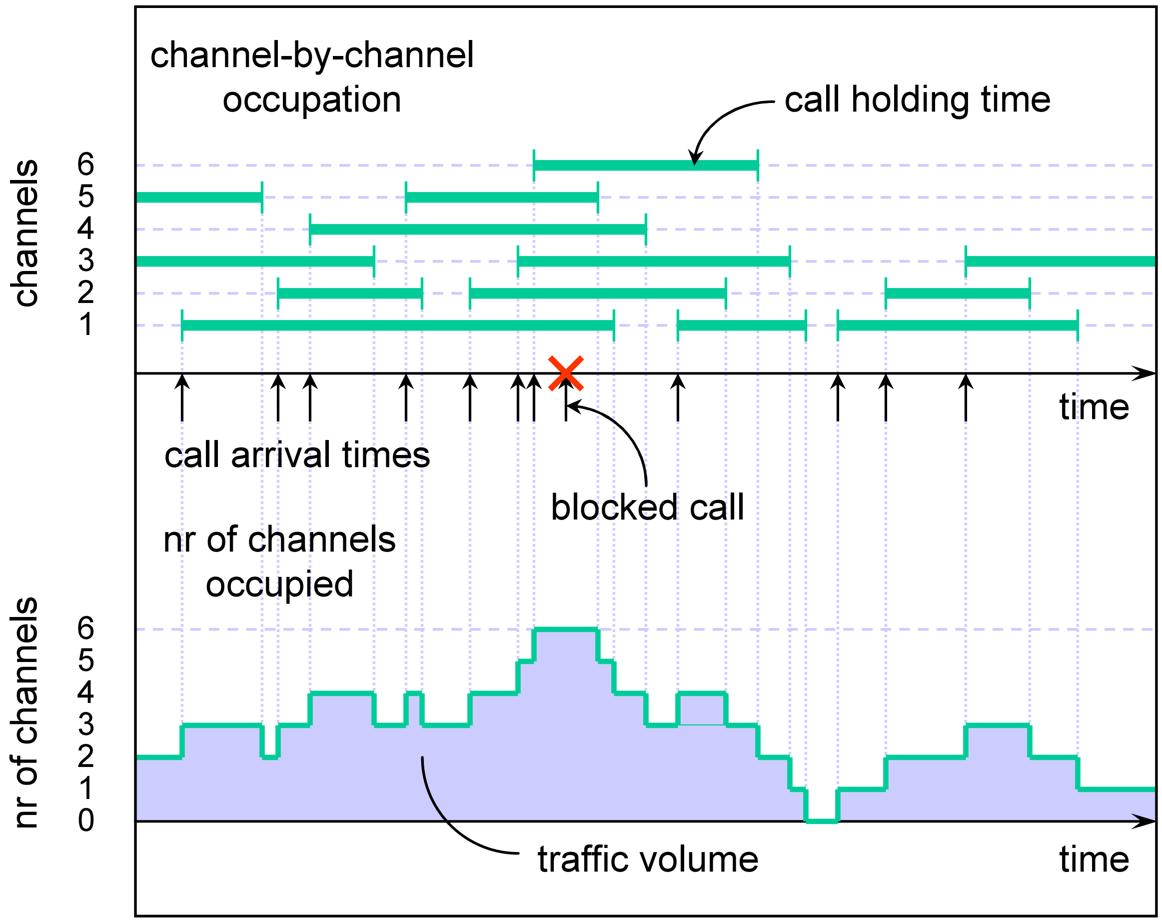


Classical model for telephone traffic (2)

- Erlang modelled this as a **pure loss system** ($m = 0$)
  - customer = call
    - λ = call arrival rate (calls per time unit)
  - service time = (call) holding time
    - $h = 1/\mu$ = average holding time (time units)
  - server = channel on the link
    - n = nr of channels on the link



Traffic process



Traffic intensity

- The strength of the offered traffic is described by the traffic intensity a
- By definition, the **traffic intensity** a is the product of the arrival rate λ and the mean holding time h:

$$a = \lambda h$$

  - The traffic intensity is a **dimensionless** quantity. Anyway, the unit of the traffic intensity a is called **erlang** (**erl**)
  - By Little's formula: traffic of one erlang means that one channel is occupied on average
- Example:
  - On average, there are 1800 new calls in an hour, and the average holding time is 3 minutes. Then the traffic intensity is

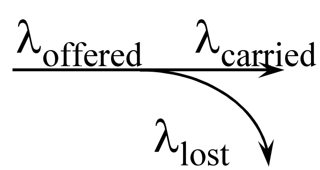
$$a = 1800 * 3 / 60 = 90 \text{ erlang}$$

Blocking

- In a loss system some calls are lost
  - a call is lost if all n channels are occupied when the call arrives
  - the term **blocking** refers to this event
- There are two different types of blocking quantities:
  - **Call blocking** B_c = probability that an arriving call finds all n channels occupied = the fraction of calls that are lost
  - **Time blocking** B_t = probability that all n channels are occupied at an arbitrary time = the fraction of time that all n channels are occupied
- The two blocking quantities are not necessarily equal
  - Example: your own mobile
  - But if calls arrive according to a Poisson process, then $B_c = B_t$
- Call blocking is a better measure for the quality of service experienced by the subscribers but, typically, time blocking is easier to calculate

Call rates

- In a loss system each call is either **lost** or **carried.** Thus, there are three types of call rates:
  - λ_{offered} = arrival rate of all call attempts
  - λ_{carried} = arrival rate of carried calls
  - λ_{lost} = arrival rate of lost calls



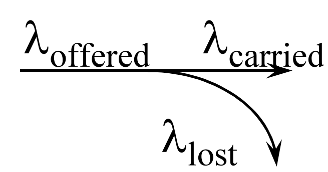
$$\lambda_{\text{offered}} = \lambda_{\text{carried}} + \lambda_{\text{lost}} = \lambda$$

$$\lambda_{\text{carried}} = \lambda(1 - B_{\text{c}})$$

$$\lambda_{\text{lost}} = \lambda B_{\text{c}}$$

Traffic streams

- The three call rates lead to the following three traffic concepts:
  - **Traffic offered** $a_{\text{offered}} = \lambda_{\text{offered}} h$
  - **Traffic carried** $a_{\text{carried}} = \lambda_{\text{carried}} h$
  - **Traffic lost** $a_{\text{lost}} = \lambda_{\text{lost}} h$



$$a_{\text{offered}} = a_{\text{carried}} + a_{\text{lost}} = a$$

$$a_{\text{carried}} = a(1 - B_{\text{c}})$$

$$a_{\text{lost}} = aB_{\text{c}}$$

- Traffic offered and traffic lost are hypothetical quantities, but traffic carried is **measurable**, since (by Little's formula) it corresponds to the average number of occupied channels on the link

Teletraffic analysis (1)

- System capacity
  - n = number of channels on the link
- Traffic load
  - a = (offered) traffic intensity
- Quality of service (from the subscribers' point of view)
  - B_c = call blocking = probability that an arriving call finds all n channels occupied
- Assume an **M/G/*n*/*n* loss system**:
  - calls arrive according to a **Poisson process** (with rate λ)
  - call holding times are independently and identically distributed according to **any distribution** with mean h

Teletraffic analysis (2)

- Then the quantitive relation between the three factors (system, traffic, and quality of service) is given by **Erlang's formula**:

$$B_{\mathrm{c}} = \mathrm{Erl}(n,a) := \frac{\frac{a^n}{n!}}{\sum_{i=0}^{n} \frac{a^i}{i!}}$$

$$n! = n \cdot (n-1) \cdot \ldots \cdot 2 \cdot 1, \quad 0! = 1$$

- Also called:
  - Erlang's B-formula
  - Erlang's blocking formula
  - Erlang's loss formula
  - Erlang's first formula

Example

- Assume that there are n = 4 channels on a link and the offered traffic is a = 2.0 erlang. Then the call blocking probability B_c is

$$B_c = \mathrm{Erl}(4,2) = \frac{\frac{2^4}{4!}}{1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!}} = \frac{\frac{16}{24}}{1 + 2 + \frac{4}{2} + \frac{8}{6} + \frac{16}{24}} = \frac{2}{21} \approx 9.5\%$$

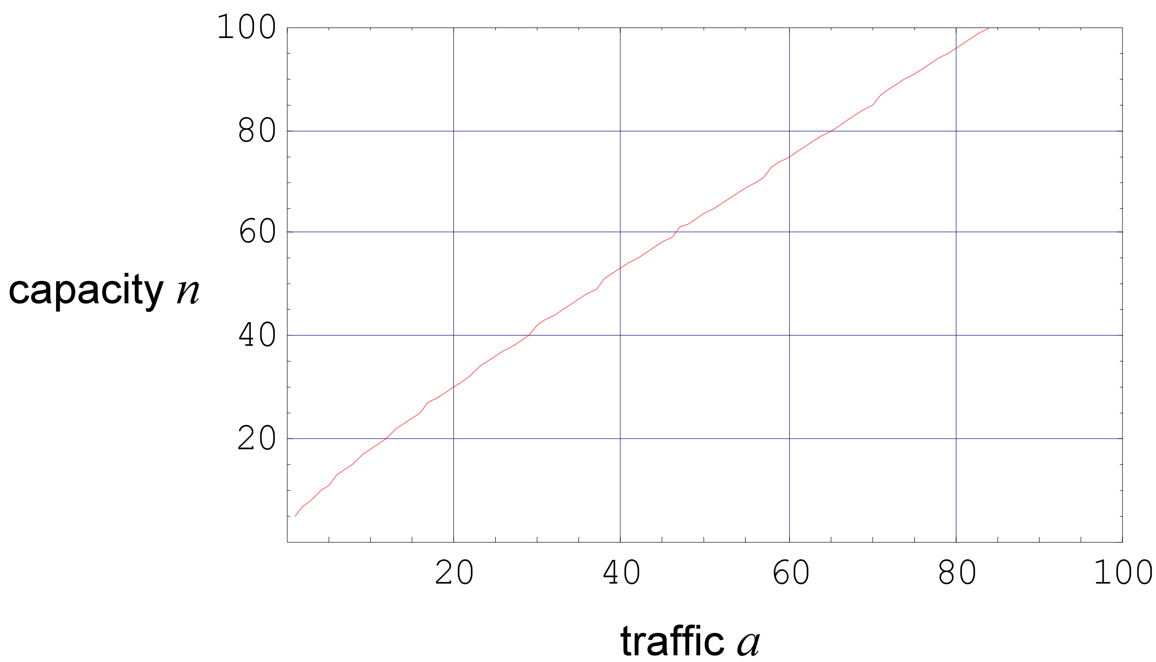
- If the link capacity is raised to n = 6 channels, then B_c reduces to

$$B_c = \mathrm{Erl}(6,2) = \frac{\frac{2^6}{6!}}{1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} + \frac{2^5}{5!} + \frac{2^6}{6!}} \approx 1.2\%$$

Capacity vs. traffic

- Given the quality of service requirement that $B_c < 1\%$, the required capacity n depends on the traffic intensity a as follows:

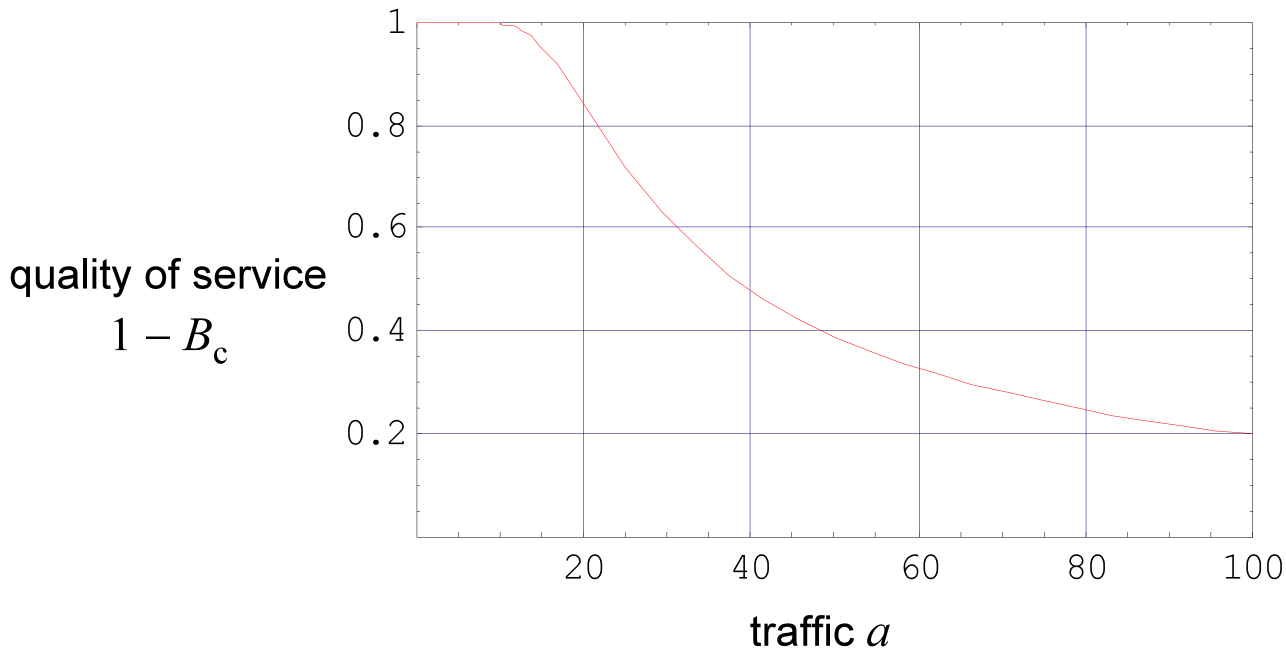
$$n(a) = \min\{i = 1,2,\ldots \mid \mathrm{Erl}(i,a) < 0.01\}$$



Quality of service vs. traffic

- Given the capacity $n = 20$ channels, the required quality of service $1 - B_c$ depends on the traffic intensity a as follows:

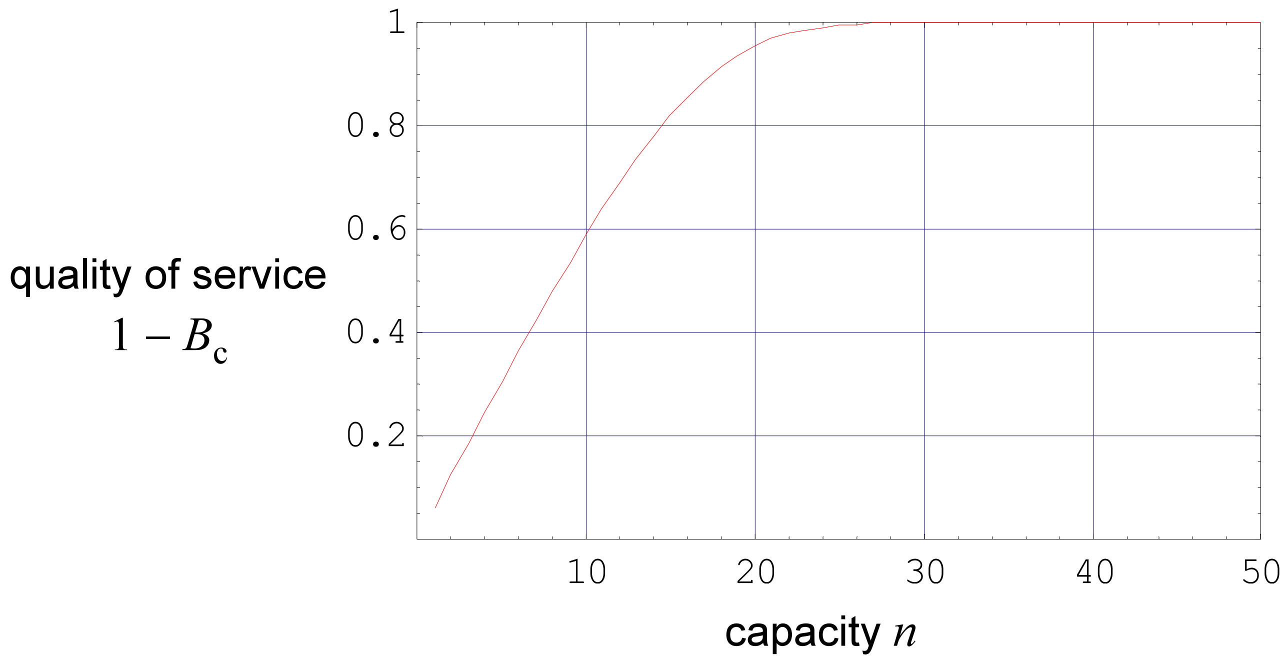
$$1 - B_c(a) = 1 - \mathrm{Erl}(20, a)$$



Quality of service vs. capacity

- Given the traffic intensity $a = 15.0$ erlang, the required quality of service $1 - B_\mathrm{c}$ depends on the capacity n as follows:

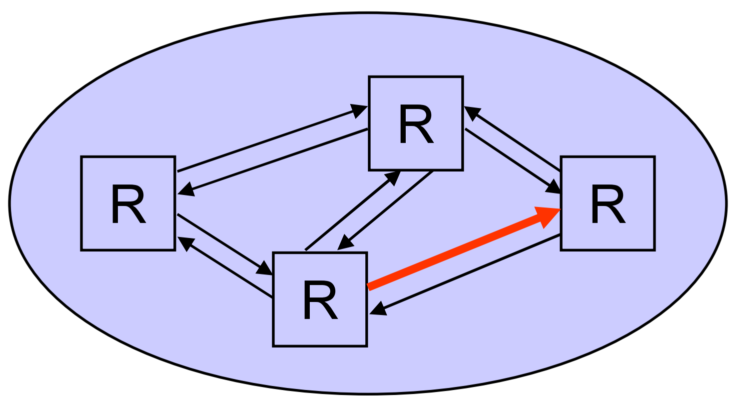
$$1 - B_\mathrm{c}(n) = 1 - \mathrm{Erl}(n, 15.0)$$



Contents

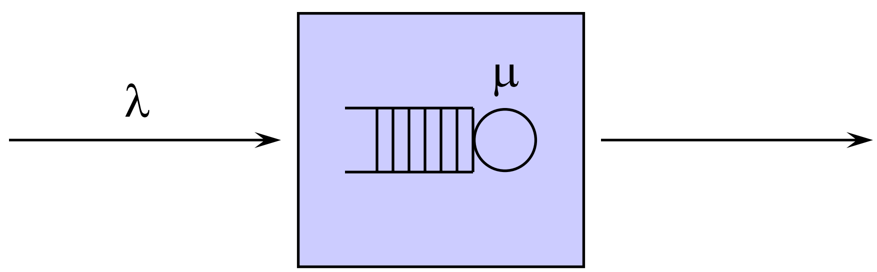
Packet level model for data traffic (1)

- **Queueing models** are suitable for describing (packet-switched) data traffic at packet level
  - Pioneering work made by many people in 60's and 70's related to ARPANET, in particular *L. Kleinrock* (`http://www.lk.cs.ucla.edu/`)
- Consider a link between two packet routers
  - traffic consists of data packets transmitted along the link

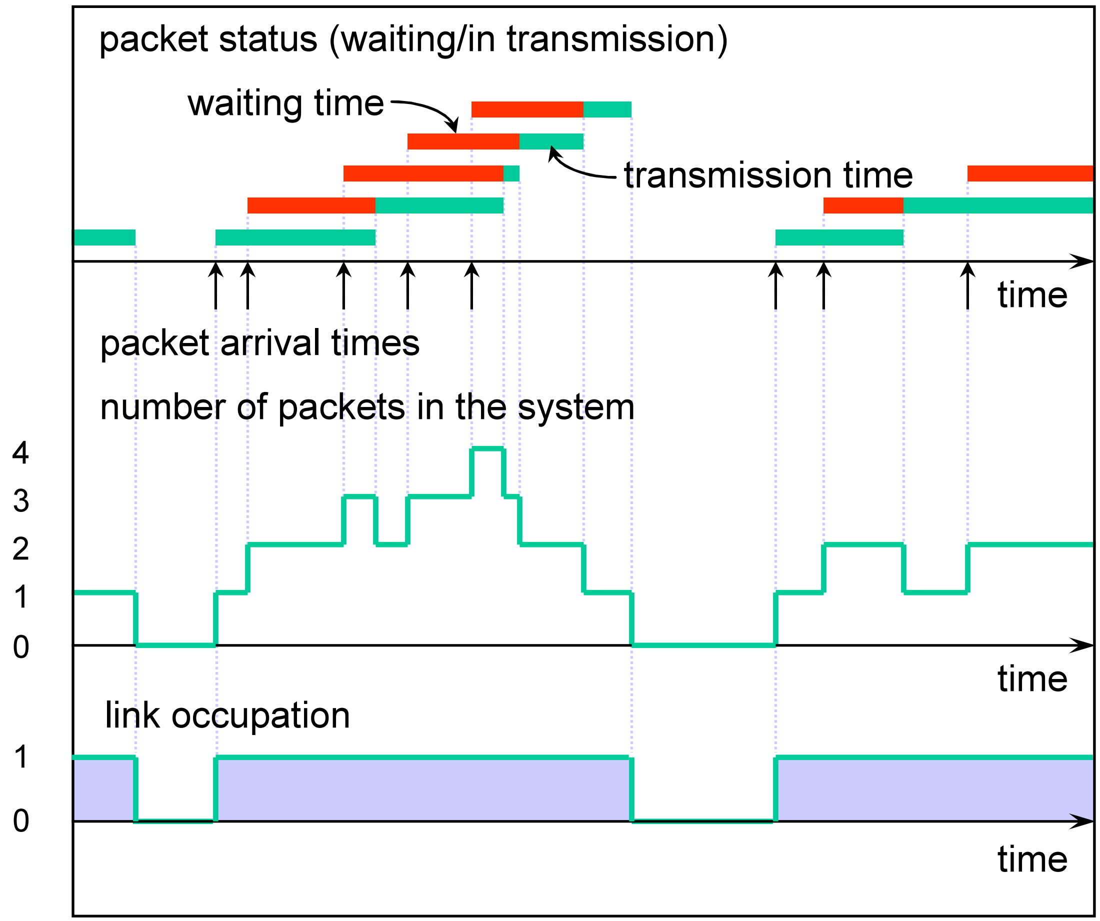


Packet level model for data traffic (2)

- This can be modelled as a **pure queueing system** with a single server ($n = 1$) and an infinite buffer ($m = \infty$)
  - customer = packet
    - λ = packet arrival rate (packets per time unit)
    - L = average packet length (data units)
  - server = link, waiting places = buffer
    - C = link speed (data units per time unit)
  - service time = packet transmission time
    - $1/\mu = L/C$ = average packet transmission time (time units)



Traffic process



Traffic load

- The strength of the offered traffic is described by the traffic load ρ
- By definition, the **traffic load** ρ is the ratio between the arrival rate λ and the service rate $\mu = C/L$:

$$\rho = \frac{\lambda}{\mu} = \frac{\lambda L}{C}$$

  - The traffic load is a **dimensionless** quantity
  - By Little's formula, it tells the **utilization factor** of the server, which is the probability that the server is busy

Example

- Consider a link between two packet routers. Assume that,
  - on average, 50,000 new packets arrive in a second,
  - the mean packet length is 1500 bytes, and
  - the link speed is 1 Gbps.
- Then the traffic load (as well as, the utilization) is

$$\rho = 50{,}000 * 1500 * 8 / 1{,}000{,}000{,}000 = 0.60 = 60\%$$

Delay

- In a queueing system, some packets have to wait before getting served
  - An arriving packet is buffered, if the link is busy upon the arrival
- **Delay** of a packet consists of
  - the **waiting time**, which depends on the state of the system upon the arrival, and
  - the **transmission time**, which depends on the length of the packet and the capacity of the link
- Example:
  - packet length = 1500 bytes
  - link speed = 1 Gbps
  - transmission time = 1500*8/1,000,000,000 = 0.000012 s = 12 μs

Teletraffic analysis (1)

- System capacity
  - C = link speed in kbps
- Traffic load
  - λ = packet arrival rate in pps (considered here as a variable)
  - L = average packet length in kbits (assumed here to be constant 1 kbit)
- Quality of service (from the users' point of view)
  - P_z = probability that a packet has to wait "too long", i.e. longer than a given reference value z (assumed here to be constant z = 0.00001 s = 10 μs)
- Assume an **M/M/1 queueing system**:
  - packets arrive according to a **Poisson process** (with rate λ)
  - packet lengths are independent and identically distributed according to the **exponential distribution** with mean L

Teletraffic analysis (2)

- Then the quantitive relation between the three factors (system, traffic, and quality of service) is given by the following formula:

$$P_z = \text{Wait}(C,\lambda;L,z) :=$$
$$\begin{cases} \frac{\lambda L}{C}\exp(-(\frac{C}{L}-\lambda)z) = \rho\exp(-\mu(1-\rho)z), & \text{if } \lambda L < C \ (\rho < 1) \\ 1, & \text{if } \lambda L \geq C \ (\rho \geq 1) \end{cases}$$

- Note:
  - The system is **stable** only in the former case ($\rho < 1$). Otherwise the number of packets in the buffer grows without limits.

Example

- Assume that packets arrive at rate $\lambda = 600{,}000$ pps $= 0.6$ packets/μs and the link speed is $C = 1.0$ Gbps $= 1.0$ kbit/μs.
- The system is stable since

$$\rho = \frac{\lambda L}{C} = 0.6 < 1$$

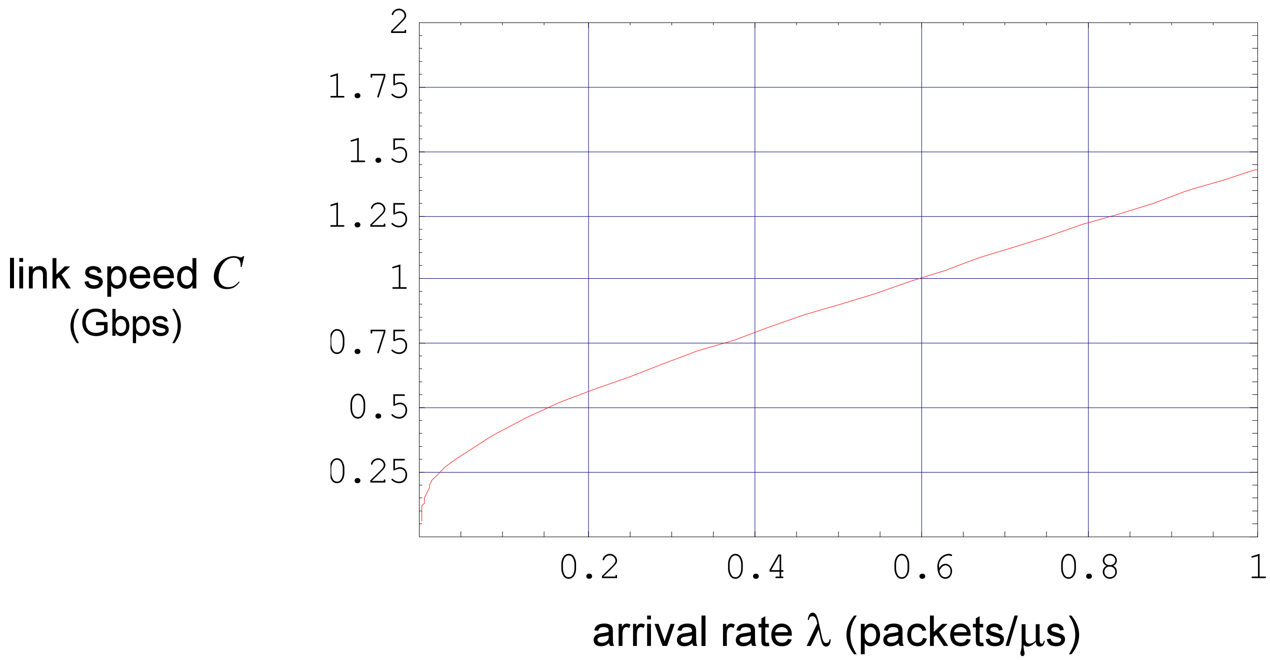
- The probability P_z that an arriving packet has to wait too long (i.e. longer than z = 10 μs) is

$$P_z = \mathrm{Wait}(1.0, 0.6; 1, 10) = 0.6 \exp(-4.0) \approx 1\%$$

Capacity vs. arrival rate

- Given the quality of service requirement that $P_z < 1\%$, the required link speed C depends on the arrival rate λ as follows:

$$C(\lambda) = \min\{c > \lambda L \mid \mathrm{Wait}(c, \lambda; 1, 10) < 0.01\}$$

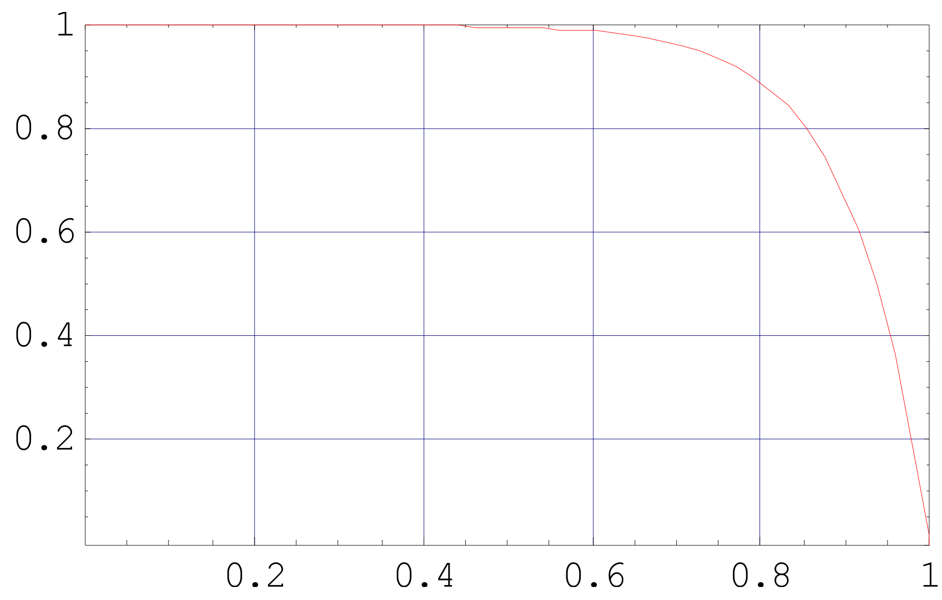


Quality of service vs. arrival rate

- Given the link speed $C = 1.0$ Gbps = 1.0 kbit/μs, the quality of service $1 - P_z$ depends on the arrival rate λ as follows:

$$1 - P_z(\lambda) = 1 - \mathrm{Wait}(1.0, \lambda; 1, 10)$$

quality of service $1 - P_z$

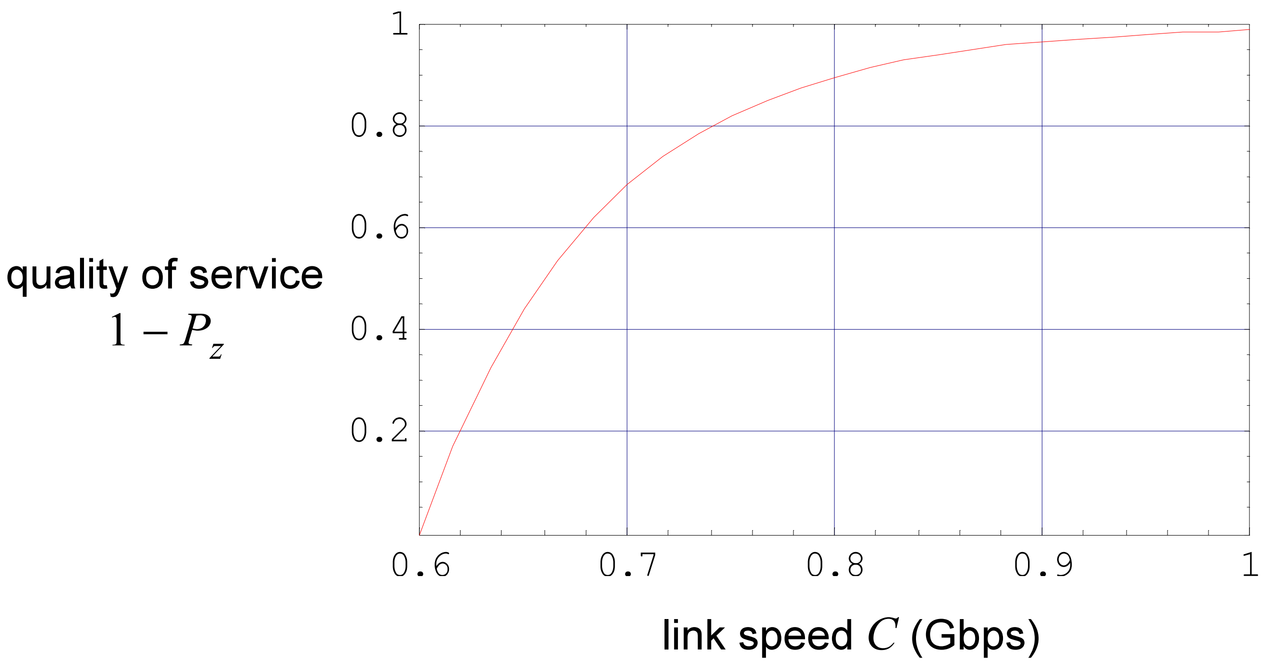


arrival rate λ (packets/μs)

Quality of service vs. capacity

- Given the arrival rate $\lambda = 600{,}000$ pps $= 0.6$ packets/μs, the quality of service $1 - P_z$ depends on the link speed C as follows:

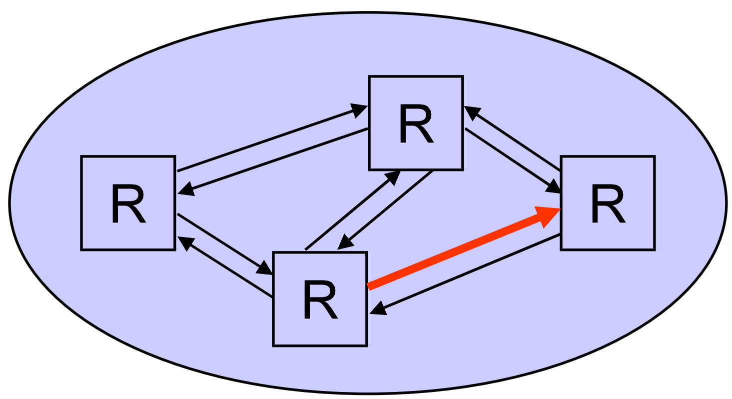
$$1 - P_z(R) = 1 - \mathrm{Wait}(C, 0.6; 1, 10)$$



Contents

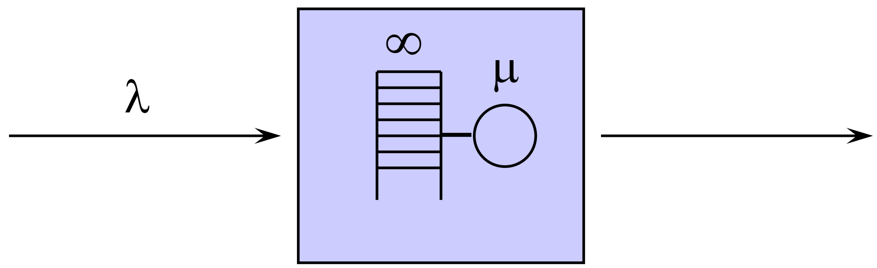
Flow level model for elastic data traffic (1)

- **Sharing models** are suitable for describing elastic data traffic at flow level
  - Elasticity refers to the adaptive sending rate of TCP flows
  - This kind of models have been proposed, e.g., by *J. Roberts* and his researchers (**http://perso.rd.francetelecom.fr/roberts/**)
- Consider a link between two packet routers
  - traffic consists of TCP flows loading the link

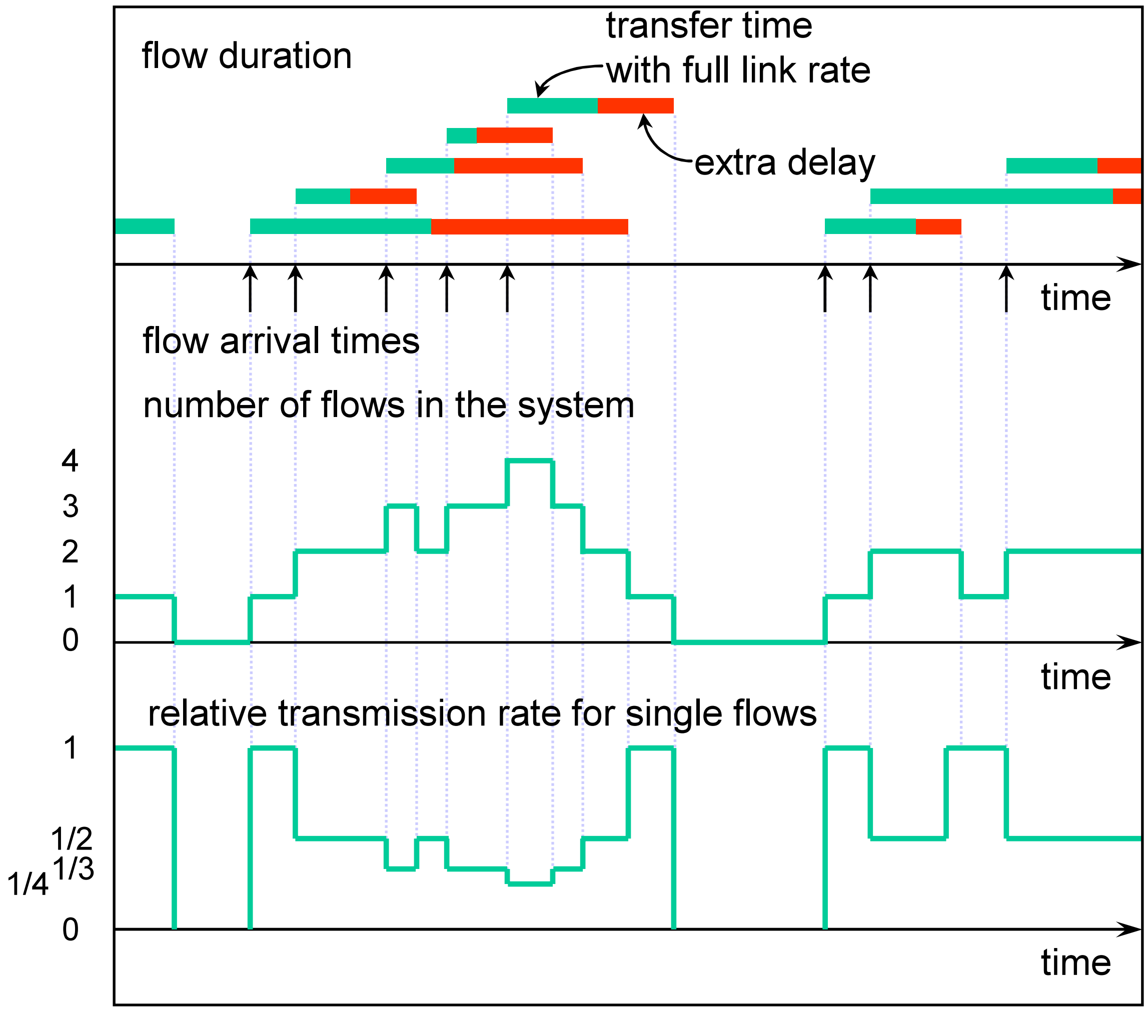


Flow level model for elastic data traffic (2)

- The simplest model is a single server ($n = 1$) **pure sharing system** with a fixed total service rate of μ
  - customer = TCP flow = file to be transferred
    - λ = flow arrival rate (flows per time unit)
    - S = average flow size = average file size (data units)
  - server = link
    - C = link speed (data units per time unit)
  - service time = file transfer time with full link speed
    - $1/\mu$ = S/C = average file transfer time with full link speed (time units)



Traffic process



Traffic load

- The strength of the offered traffic is described by the traffic load ρ
- By definition, the **traffic load** ρ is the ratio between the arrival rate λ and the service rate $\mu = C/S$:

$$\rho = \frac{\lambda}{\mu} = \frac{\lambda S}{C}$$

  - The traffic load is (again) a dimensionless quantity
  - It tells the utilization factor of the server

Example

- Consider a link between two packet routers. Assume that,
  - on average, 50 new flows arrive in a second,
  - average flow size is 1,500,000 bytes, and
  - link speed is 1 Gbps.
- Then the traffic load (as well as, the utilization) is

$$\rho = 50 * 1{,}500{,}000 * 8 / 1{,}000{,}000{,}000 = 0.60 = 60\%$$

Throughput

- In a sharing system the service capacity is shared among all active flows. It follows that **all** flows get delayed (unless there is only a single active flow)
- By definition, the ratio between the average flow size S and the average total delay D of a flow is called **throughput** θ,

$$\theta = S / D$$

- Example:
  - $S = 1$ Mbit
  - $D = 5$ s
  - $\theta = S/D = 0.2$ Mbps

Teletraffic analysis (1)

- System capacity
  - C = link speed in Mbps
- Traffic load
  - λ = flow arrival rate in flows per second (considered here as a variable)
  - S = average flow size in kbits (assumed here to be constant 1 Mbit)
- Quality of service (from the users' point of view)
  - θ = throughput
- Assume an **M/G/1-PS sharing system**:
  - flows arrive according to a **Poisson process** (with rate λ)
  - flow sizes are independent and identically distributed according to **any distribution** with mean S

Teletraffic analysis (2)

- Then the quantitive relation between the three factors (system, traffic, and quality of service) is given by the following formula:

$$\theta = \mathrm{Xput}(C, \lambda; S) := \begin{cases} C - \lambda S = C(1-\rho), & \text{if } \lambda S < C \ (\rho < 1) \\ 0, & \text{if } \lambda S \geq C \ (\rho \geq 1) \end{cases}$$

- Note:
  - The system is **stable** only in the former case ($\rho < 1$). Otherwise the number of flows as well as the average delay grows without limits. In other words, the throughput of a flow goes to zero.

Example

- Assume that flows arrive at rate $\lambda = 600$ flows per second and the link speed is $C = 1000$ Mbps $= 1.0$ Gbps.
- The system is stable since

$$\rho = \frac{\lambda S}{C} = \frac{600}{1000} = 0.6 < 1$$

- Throughput is

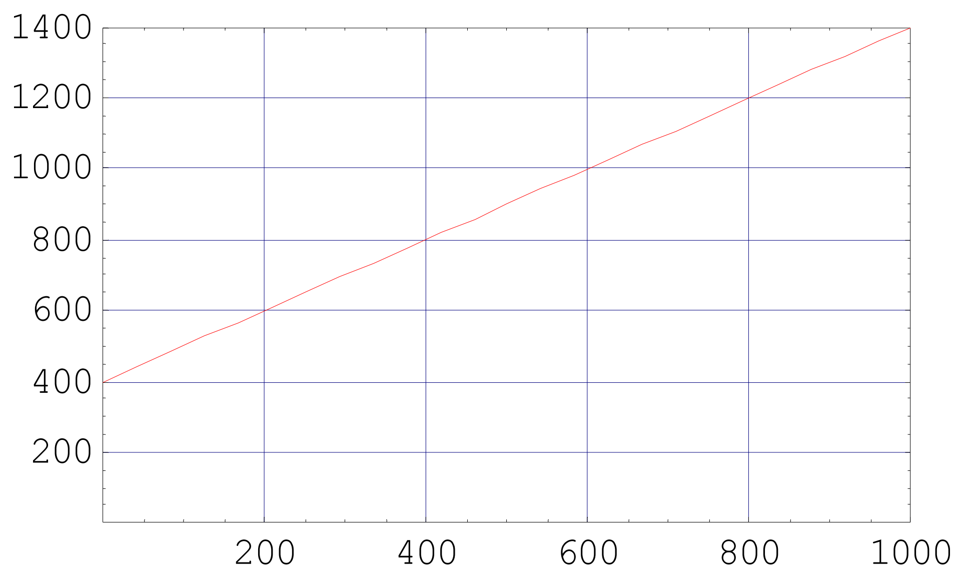
$$\theta = \mathrm{Xput}(1000, 600; 1) = 1000 - 600 = 400\ \mathrm{Mbps} = 0.4\ \mathrm{Gbps}$$

Capacity vs. arrival rate

- Given the quality of service requirement that $\theta \geq 400$ Mbps, the required link speed C depends on the arrival rate λ as follows:

$$C(\lambda) = \min\{c > \lambda S \mid \text{Xput}(c, \lambda; 1) \geq 400\} = \lambda S + 400$$

link speed C
(Mbps)

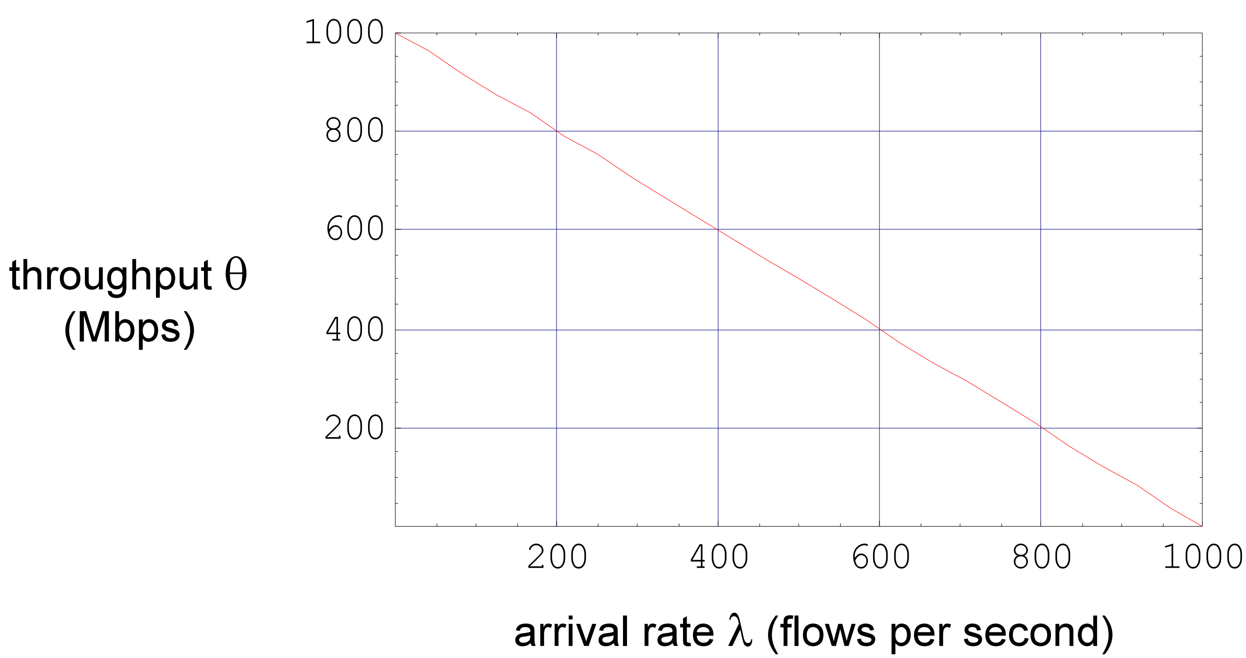


arrival rate λ (flows per second)

Quality of service vs. arrival rate

- Given the link speed $C = 1000$ Mbps, the quality of service θ depends on the arrival rate λ as follows:

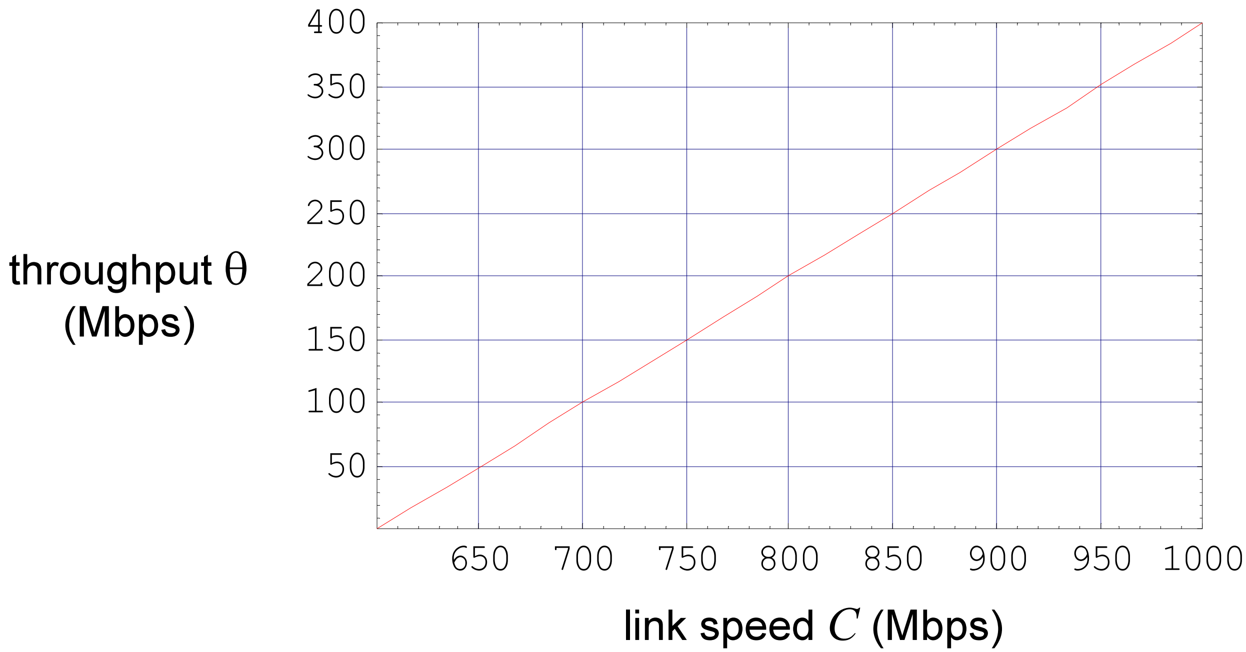
$$\theta(\lambda) = \text{Xput}(1000, \lambda; 1) = 1000 - \lambda S, \quad \lambda < 1000/S$$



Quality of service vs. capacity

- Given the arrival rate $\lambda = 600$ flows per second, the quality of service θ depends on the link speed C as follows:

$$\theta(C) = \mathrm{Xput}(C,600;1) = C - 600S, \quad C > 600S$$

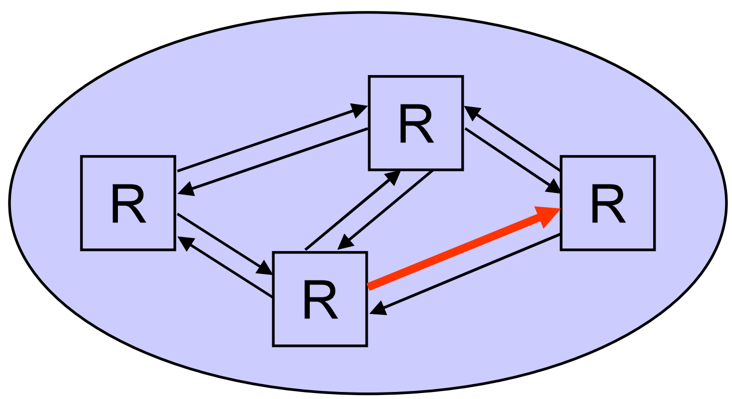


Contents

- Model for telephone traffic
- Packet level model for data traffic
- Flow level model for elastic data traffic
- Flow level model for streaming data traffic

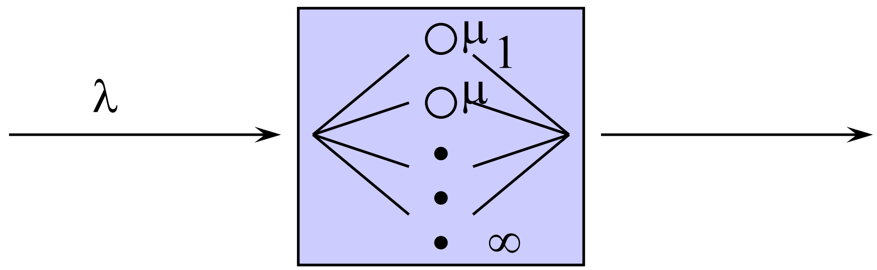
Flow level model for streaming CBR traffic (1)

- **Infinite system** is suitable for describing streaming CBR traffic at flow level
  - The transmission rate and flow duration of a streaming flow are insensitive to the network state
  - This kind of models applied in 90's to the teletraffic analysis of CBR traffic in ATM networks
- Consider a link between two packet routers
  - traffic consists of UDP flows carrying CBR traffic (like VoIP) and loading the link

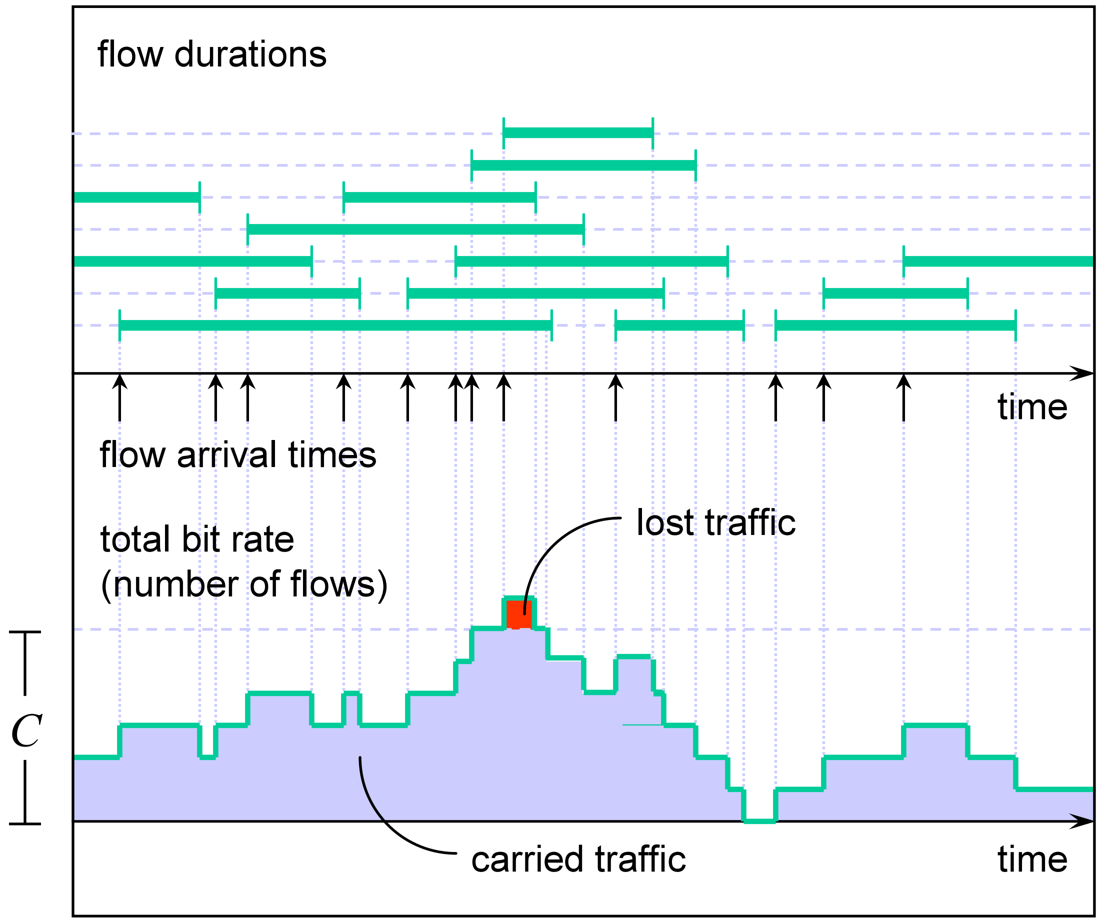


Flow level model for streaming CBR traffic (2)

- Model: an **infinite system** ($n = \infty$)
  - customer = UDP flow = CBR bit stream
    - λ = flow arrival rate (flows per time unit)
  - service time = flow duration
    - $h = 1/\mu$ = average flow duration (time units)
- **Bufferless** flow level model:
  - when the total transmission rate of the flows exceeds the link capacity, bits are lost (uniformly from all flows)



Traffic process



Offered traffic

- Let r denote the bit rate of any flow
- The strength of offered traffic is described by average total bit rate R
  - By Little's formula, the average number of flows is

$$a = \lambda h$$

  - This may be called **traffic intensity** (cf. telephone traffic)
  - It follows that

$$R = ar = \lambda hr$$

Loss ratio

- Let N denote the number of flows in the system
- When the total transmission rate Nr exceeds the link capacity C, bits are lost with rate

$$Nr - C$$

- The average loss rate is thus

$$E[(Nr - C)^+] = E[\max\{Nr - C, 0\}]$$

- By definition, the **loss ratio** p_{loss} gives the ratio between the traffic lost and the traffic offered:

$$p_{\text{loss}} = \frac{E[(Nr - C)^+]}{E[Nr]} = \frac{1}{ar} E[(Nr - C)^+]$$

Teletraffic analysis (1)

- System capacity
  - $C = nr$ = link speed in kbps
- Traffic load
  - $R = ar$ = offered traffic in kbps
  - r = bit rate of a flow in kbps.
- Quality of service (from the users' point of view)
  - p_{loss} = loss ratio
- Assume an **M/G/∞ infinite system**:
  - flows arrive according to a **Poisson process** (with rate λ)
  - flow durations are independent and identically distributed according to **any distribution** with mean h

Teletraffic analysis (2)

- Then the quantitive relation between the three factors (system, traffic, and the quality of service) is given by the following formula

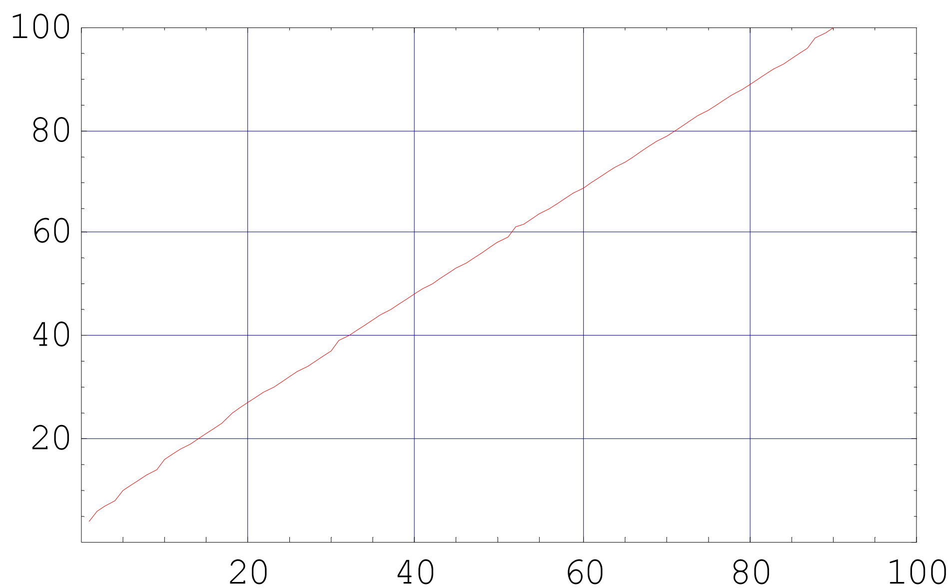
$$p_{\text{loss}} = \text{LR}(n,a) := \frac{1}{a} \sum_{i=n+1}^{\infty} (i-n) \frac{a^i}{i!} e^{-a}$$

- Example:
  - $n = 20$
  - $a = 14.36$
  - $p_{\text{loss}} = 0.01$

Capacity vs. traffic

- Given the quality of service requirement that $p_{\text{loss}} < 1\%$, the required capacity n depends on the traffic intensity a as follows:

$$n(a) = \min\{i = 1,2,\dots \mid \mathrm{LR}(i,a) < 0.01\}$$



capacity n

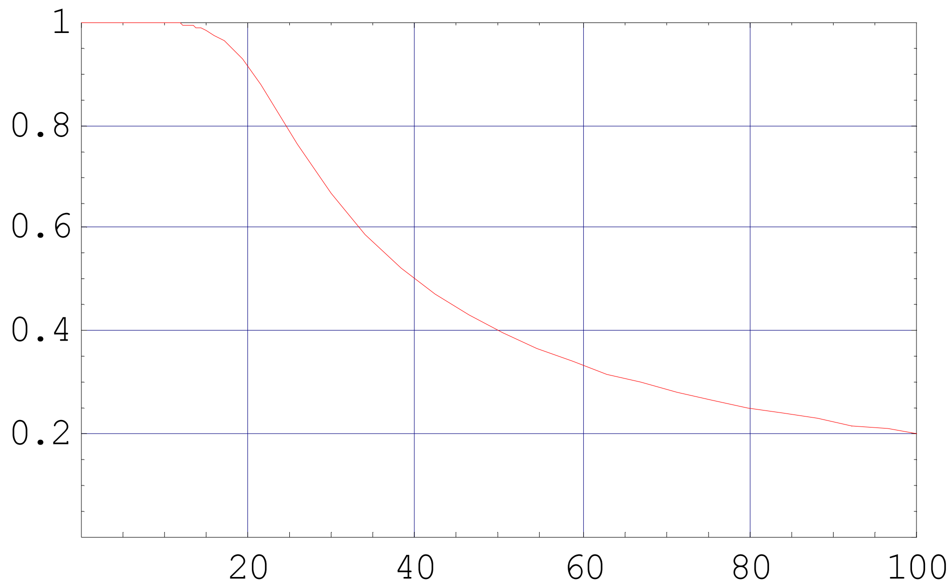
traffic a

Quality of service vs. traffic

- Given the capacity $n = 20$, the required quality of service $1 - p_{\text{loss}}$ depends on the traffic intensity a as follows:

$$1 - p_{\text{loss}}(a) = 1 - \mathrm{LR}(20, a)$$

quality of service $1 - p_{\text{loss}}$

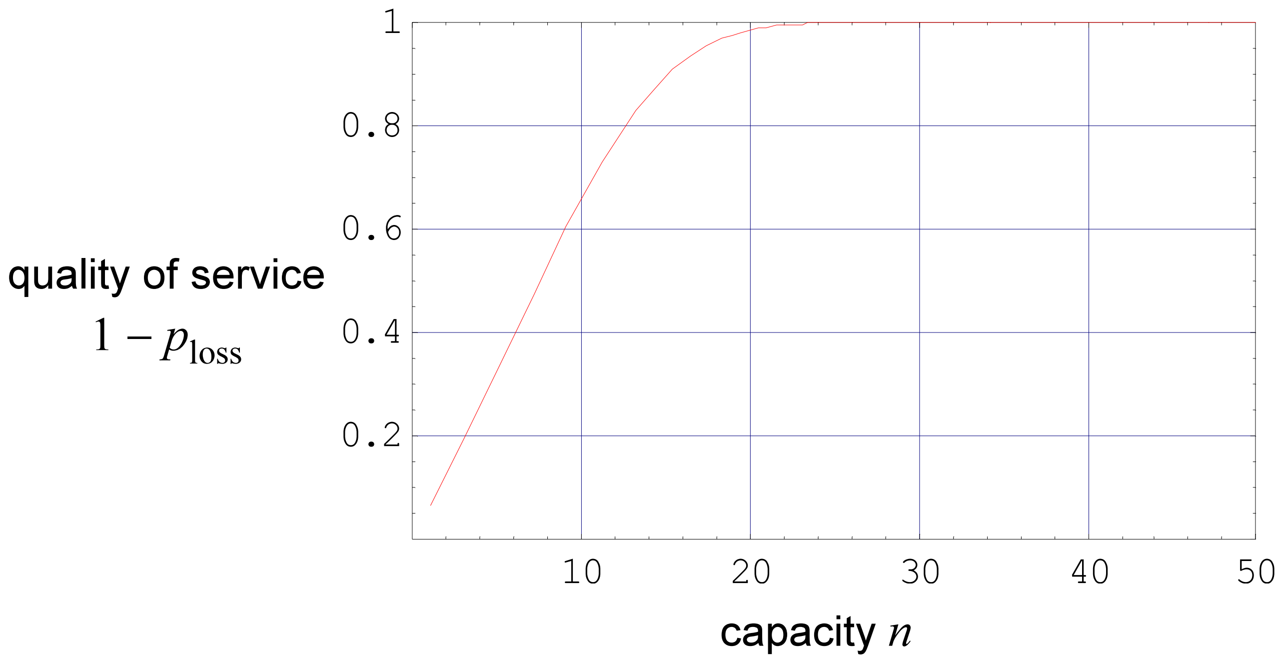


traffic a

Quality of service vs. capacity

- Given the traffic intensity $a = 15.0$ erlang, the required quality of service $1 - p_{\text{loss}}$ depends on the capacity n as follows:

$$1 - p_{\text{loss}}(n) = 1 - \text{LR}(n, 15.0)$$



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