

Problems 2, 5 and 6 are homework exercises. Return your answers into the course box of the laboratory (G-wing, 2. floor) latest at 10.00 on Tuesday 7.2.

1. *Demo*

Buses are leaving from a bus stop regularly in every 15 minutes. Taxis pass by according to a Poisson process with rate once in 15 minutes. You arrive at the bus stop at a random time instant.

- (a) What is your expected waiting time until the first bus arrives?
- (b) What is your expected waiting time until the first taxi arrives?
- (c) What is the probability that you have to wait longer than 10 minutes before the first taxi or bus passes by?

2. *Homework exercise*

A link in a packet switched network carries on average 10 packets/s. Assume that packets arrive according to a Poisson process. Each packet is an acknowledgement (ACK) independently of other packets with probability 10 %. Consider a random time interval of length one second.

- a) What is the probability that there is at least one ACK packet on the link during the interval?
- b) What is the expected total number of packets if four ACK packets were observed during the interval?

3. *Demo*

Connection requests arrive at a server according to a Poisson process with intensity λ . If the server is overloaded, its throughput collapses quickly. To prevent such incidents, the server implements a congestion control scheme based on gapping. In this scheme after every accepted request the server refuses to accept any new connections during the following time period of length T . Assume that requests arriving during the gap interval are just discarded and they are not renewed.

- a) How many requests are accepted in a time unit on average?
- b) Determine the rate of accepted requests in the extreme cases when T is very large or very small, respectively.

4. *Demo*

The Ehrenfest model was used, at a time, to shed light on a paradox related to the second law of thermodynamics. The model can be described as follows. A closed system is constructed from K randomly moving gas molecules and two containers which are connected so that each of the molecules changes the container at rate λ , independently of the other molecules. Let $X(t)$ denote the number of molecules in one of the containers.

- a) Draw the state transition diagram of Markov process $X(t)$.
- b) Derive the equilibrium distribution of $X(t)$.
- c) Compare the conditional probabilities $P\{X(t) = K \mid X(0) = \frac{K}{2}\}$ and $P\{X(t) = \frac{K}{2} \mid X(0) = K\}$ (K assumed to be even), when t is large.

5. *Homework exercise*

A Markov process is defined in the state space $\{0, 1, 2, 3\}$ with the state transitions rates q_{ij} collected in the transition matrix $Q = (q_{ij} \mid i, j = 0, 1, 2, 3)$, where $q_{ii} = -q_i$ for all i , as follows:

$$Q = \begin{pmatrix} -3 & 2 & 0 & 1 \\ 1 & -3 & 2 & 0 \\ 0 & 1 & -3 & 2 \\ 2 & 0 & 1 & -3 \end{pmatrix}$$

- (a) Draw the state transition diagram of Markov process $X(t)$.
- (b) Derive the equilibrium distribution of $X(t)$.

6. *Homework exercise*

A Markov process is defined in the state space $\{0, 1, 2, 3\}$ with the state transitions rates q_{ij} collected in the transition matrix $Q = (q_{ij} \mid i, j = 0, 1, 2, 3)$, where $q_{ii} = -q_i$ for all i , as follows:

$$Q = \begin{pmatrix} -6 & 1 & 2 & 3 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

- (a) Draw the state transition diagram of Markov process $X(t)$.
- (b) Derive the equilibrium distribution of $X(t)$.